## Newton's Laws

## The Physics Classroom Tutorial

http://www.physicsclassroom.com/

## Newton's Laws

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Newton's Laws - Lesson 1 - Newton's First Law of Motion

## Newton's First Law

In a previous chapter of study, the variety of ways by which motion can be described (words, graphs, diagrams, numbers, etc.) was discussed. In this unit (Newton's Laws of Motion), the ways in which motion can be explained will be discussed. Isaac Newton (a 17th century scientist) put forth a variety of laws that explain why objects move (or don't move) as they do. These three laws have become known as Newton's three laws of motion. The focus of Lesson 1 is Newton's first law of motion - sometimes referred to as the law of inertia.

Newton's first law of motion is often stated as
An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

## Two Clauses and a Condition

There are two clauses or parts to this statement - one that predicts the behavior of stationary objects and the other that predicts the behavior of moving objects. The two parts are summarized in the following diagram.


The behavior of all objects can be described by saying that objects tend to "keep on doing what they're doing" (unless acted upon by an unbalanced force). If at rest, they will continue in this same state of rest. If in motion with an eastward velocity of $5 \mathrm{~m} / \mathrm{s}$, they will continue in this same
 state of motion ( $5 \mathrm{~m} / \mathrm{s}$, East). If in motion with a leftward velocity of $2 \mathrm{~m} / \mathrm{s}$, they will continue in this same state of motion ( $2 \mathrm{~m} / \mathrm{s}$, left). The state of motion of an object is maintained as long as the object is not acted upon by an unbalanced force. All objects resist changes in their state of motion - they tend to "keep on doing what they're doing."

There is an important condition that must be met in order for the first law to be applicable to any given motion. The condition is described by the phrase "... unless acted upon by an unbalanced force." As the long as the forces are not unbalanced - that is, as long as the forces are balanced the first law of motion applies. This concept of a balanced versus and unbalanced force will be discussed in more detail later in Lesson 1.

Suppose that you filled a baking dish to the rim with water and walked around an oval track making an attempt to complete a lap in the least amount of time. The water would have a tendency to spill from the container during specific locations on the track. In general the water spilled when:

- the container was at rest and you attempted to move it
- the container was in motion and you attempted to stop it
- the container was moving in one direction and you attempted to change its direction.

The water spills whenever the state of motion of the container is changed. The water resisted this change in its own state of motion. The water tended to "keep on doing what it was doing." The container was moved from rest to a high speed at the starting line; the water remained at rest and
 spilled onto the table. The container was stopped near the finish line; the water kept moving and spilled over container's leading edge. The container was forced to move in a different direction to make it around a curve; the water kept moving in the same direction and spilled over its edge. The behavior of the water during the lap around the track can be explained by Newton's first law of motion.

## Everyday Applications of Newton's First Law

There are many applications of Newton's first law of motion. Consider some of your experiences in an automobile. Have you ever observed the behavior of coffee in a coffee cup filled to the rim while starting a car from rest or while bringing a car to rest from a state of motion? Coffee "keeps on doing what it is doing." When you accelerate a car from rest, the road provides an unbalanced force on the spinning wheels to push the car forward; yet the coffee (that was at rest) wants to stay at rest. While the car accelerates forward, the coffee remains in the same position; subsequently, the car accelerates out from under the coffee and the coffee spills in your lap. On the other hand, when braking from a state of motion the coffee continues forward with the same speed and in the same direction, ultimately hitting the windshield or the dash. Coffee in motion stays in motion.

Have you ever experienced inertia (resisting changes in your state of motion) in an automobile while it is braking to a stop? The force of the road on the locked wheels provides the unbalanced force to change the car's state of motion, yet there is no unbalanced force to change your own state of motion. Thus, you continue in motion, sliding along the seat in forward motion. A
 person in motion stays in motion with the same speed and in the same direction ... unless acted upon by the unbalanced force of a seat belt. Yes! Seat belts are used to provide safety for passengers whose motion is governed by Newton's laws. The seat belt provides the unbalanced force that brings you from a state of motion to a state of rest. Perhaps you could speculate what would occur when no seat belt is used.

## Animation

There are many more applications of Newton's first law of motion. Several applications are listed below. Perhaps you could think about the law of inertia and provide explanations for each application.

- Blood rushes from your head to your feet while quickly stopping when riding on a descending elevator.
- The head of a hammer can be tightened onto the wooden handle by banging the bottom of the handle against a hard surface.
- A brick is painlessly broken over the hand of a physics teacher by slamming it with a hammer. (CAUTION: do not attempt this at home!)
- To dislodge ketchup from the bottom of a ketchup bottle, it is often turned upside down and thrusted downward at high speeds and then
 abruptly halted.
- Headrests are placed in cars to prevent whiplash injuries during rear-end collisions.
- While riding a skateboard (or wagon or bicycle), you fly forward off the board when hitting a curb or rock or other object that abruptly halts the motion of the skateboard.


## Animation

## Try This

Acquire a metal coat hanger for which you have permission to destroy. Pull the coat hanger apart. Using duct tape, attach two tennis balls to opposite ends of the coat hanger as shown in the diagram at the right. Bend the hanger so that there is a flat part
 that balances on the head of a person. The ends of the hanger with the tennis balls should hang low (below the balancing point). Place the hanger on your head and balance it. Then quickly spin in a circle. What do the tennis balls do?

## Inertia and Mass

Newton's first law of motion states that "An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force." Objects tend to "keep on doing what they're doing." In fact, it is the natural tendency of objects to resist changes in their state of motion. This tendency to resist changes in their state of motion is described as inertia.
Inertia: the resistance an object has to a change in its state of motion.

Newton's conception of inertia stood in direct opposition to more popular conceptions about motion. The dominant thought prior to Newton's day was that it was the natural tendency of objects to come to a rest position. Moving objects, so it was believed, would eventually stop moving; a force was necessary to keep an object moving. But if left to itself, a moving object would eventually come to rest and an object at rest would stay at rest; thus, the idea that dominated people's thinking for nearly 2000 years prior to Newton was that it was the natural tendency of all objects to assume a rest position.

## Galileo and the Concept of Inertia

Galileo, a premier scientist in the seventeenth century, developed the concept of inertia. Galileo reasoned that moving objects eventually stop because of a force called friction. In experiments using a pair of inclined planes facing each other, Galileo observed that a ball would roll down one plane and up the opposite plane to approximately the same height. If smoother planes were used, the ball would roll up the opposite plane even closer to the original height. Galileo reasoned that any difference between initial and final heights was due to the presence of friction. Galileo postulated that if friction could be entirely eliminated, then the ball would reach exactly the same height.

Galileo further observed that regardless of the angle at which the planes were oriented, the final height was almost always equal to the initial height. If the slope of the opposite incline were reduced, then the ball would roll a further distance in order to reach that original height.

## If friction could be eliminated...



Initial height equals
final height


As the angle of the opposing incline is reduced, the ball must roll even a farther distance in order to attain the original height. What happens if the opposing incline is not inclined?

Galileo's reasoning continued - if the opposite incline were elevated at nearly a 0 -degree angle, then the ball would roll almost forever in an effort to reach the original height. And if the opposing incline was not even inclined at all (that is, if it were oriented along the horizontal), then ... an object in motion would continue in motion... .

If friction could be eliminated...


If a ball stops $r$ hen it attains its original height, then this ball would never stop. It would roll forever if friction were absent.

## Forces Don't Keep Objects Moving

Isaac Newton built on Galileo's thoughts about motion. Newton's first law of motion declares that a force is not needed to keep an object in motion. Slide a book across a table and watch it slide to a rest position. The book in motion on the table top does not come to a rest position


As a book slides across a table from left to right, the force of friction acts on the book to sloy it down and bring it to rest. because of the absence of a force; rather it is the presence of a force - that force being the force of friction - that brings the book to a rest position. In the absence of a force of friction, the book would continue in motion with the same speed and direction - forever! (Or at least to the end of the table top.) A force is not required to keep a moving book in motion. In actuality, it is a force that brings the book to rest.


## Mass as a Measure of the Amount of Inertia

All objects resist changes in their state of motion. All objects have this tendency - they have inertia. But do some objects have more of a tendency to resist changes than others? Absolutely yes! The tendency of an object to resist changes in its state of motion varies with mass. Mass is that quantity that is solely dependent upon the inertia of an object. The more inertia that an object has, the more mass that it has. A more
 massive object has a greater tendency to resist changes in its state of motion.

Suppose that there are two seemingly identical bricks at rest on the physics lecture table. Yet one brick consists of mortar and the other brick consists of Styrofoam. Without lifting the bricks, how could you tell which brick was the Styrofoam brick? You could give the bricks an identical push in an effort to change their state of motion. The brick that offers the least resistance is the brick with the least inertia - and therefore the brick with the least mass (i.e., the Styrofoam brick).

A common physics demonstration relies on this principle that the more massive the object, the more that object resist changes in its state of motion. The demonstration goes as follows: several massive books are placed upon a teacher's head. A wooden board is placed on top of the books and a hammer is used to drive a nail into the board. Due to the large mass of the books, the force of the hammer is sufficiently resisted (inertia). This is demonstrated by the fact that the teacher does not feel the hammer blow. (Of course, this story may explain many of the observations that you previously have made concerning your "weird
 physics teacher.") A common variation of this demonstration involves breaking a brick over the teacher's hand using the swift blow of a hammer. The massive bricks resist the force and the hand is not hurt. (CAUTION: do not try these demonstrations at home.)

## Watch It!

A physics instructor explains the property of inertia using a phun physics demonstration. http://www.youtube.com/watch?feature=player_detailpage\&v=--MDILG7Znk (youtube inertia demo)

## Check Your Understanding

1. Imagine a place in the cosmos far from all gravitational and frictional influences. Suppose that you visit that place (just suppose) and throw a rock. The rock will:
a. gradually stop.
b. continue in motion in the same direction at constant speed.


## Explain.

2. A $2-\mathrm{kg}$ object is moving horizontally with a speed of $4 \mathrm{~m} / \mathrm{s}$. How much net force is required to keep the object moving at this speed and in this direction?

## Explain.

3. Mac and Tosh are arguing in the cafeteria. Mac says that if he flings the Jell-O with a greater speed it will have a greater inertia. Tosh argues that inertia does not depend upon speed, but rather upon mass. Who do you agree with? Explain why.
4. Supposing you were in space in a weightless environment, would it require a force to set an object in motion? Explain.
5. Fred spends most Sunday afternoons at rest on the sofa, watching pro football games and consuming large quantities of food. What affect (if any) does this practice have upon his inertia?

## Explain.

6. Ben Tooclose is being chased through the woods by a bull moose that he was attempting to photograph. The enormous mass of the bull moose is extremely intimidating. Yet, if Ben makes a zigzag pattern through the woods, he will be able to use the large mass of the moose to his own advantage. Explain this in terms of inertia and Newton's first law of motion.
7. Two bricks are resting on edge of the lab table. Shirley Sheshort stands on her toes and spots the two bricks. She acquires an intense desire to know which of the two bricks are most massive. Since Shirley is vertically challenged, she is unable to reach high enough and lift the bricks; she can however reach high enough to give the bricks a push. Discuss how the process of pushing the bricks will allow Shirley to determine which of the two bricks is most massive. What difference will Shirley observe and how can this observation lead to the necessary conclusion?

## State of Motion

Inertia is the tendency of an object to resist changes in its state of motion. But what is meant by the phrase state of motion? The state of motion of an object is defined by its velocity - the speed with a direction. Thus, inertia could be redefined as follows:

Inertia: tendency of an object to resist changes in its velocity.

An object at rest has zero velocity - and (in the absence of an unbalanced force) will remain with a zero velocity. Such an object will not change its state of motion (i.e., velocity) unless acted upon by an unbalanced force. An object in motion with a velocity of $2 \mathrm{~m} / \mathrm{s}$, East will (in the absence of an unbalanced force) remain in motion with a velocity of $2 \mathrm{~m} / \mathrm{s}$, East. Such an object will not change its state of motion (i.e., velocity) unless acted upon by an unbalanced force. Objects resist changes in their velocity.

As learned in an earlier unit, an object that is not changing its velocity is said to have an acceleration of $0 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. Thus, we could provide an alternative means of defining inertia:

## Inertia: tendency of an object to resist acceleration.

## Watch It!

An air track glider is shown moving across an air track. Air is blown through many small holes in the track in order to lift the glider off the track. This reduces, maybe even eliminates, the action of surface friction upon the glider. The glider moves with what seems to be a constant speed motion. As they say: objects in motion stay in motion ... .
$\underline{h t t p: / / w w w . y o u t u b e . c o m / w a t c h ? f e a t u r e=p l a y e r \_e m b e d d e d \& v=V 366 m K D P x g s ~}$
(Newton's First Law.MP4)

## Check Your Understanding

1. A group of physics teachers is taking some time off for a little puttputt golf. The 15th hole at the Hole-In-One Putt-Putt Golf Course has a large metal rim that putters must use to guide their ball towards the hole. Mr. S guides a golf ball around the metal rim When the ball leaves the rim, which path (1, 2, or 3 ) will the golf ball follow? Explain.


## Animation

2. A $4.0-\mathrm{kg}$ object is moving across a friction-free surface with a constant velocity of $2 \mathrm{~m} / \mathrm{s}$. Which one of the following horizontal forces is necessary to maintain this state of motion? Explain your answer.
a. 0 N
b. 0.5 N
c. 2.0 N
d. 8.0 N
e. depends on the speed

## Balanced and Unbalanced Forces

Newton's first law of motion has been frequently stated throughout this lesson.
An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

## Balanced Forces

But what exactly is meant by the phrase unbalanced force? What is an unbalanced force? In pursuit of an answer, we will first consider a physics book at rest on a tabletop. There are two forces acting upon the book. One force - the Earth's gravitational pull - exerts a downward force. The other force - the push of the table on the book (sometimes referred to as a normal force) pushes upward on the book.

The forces on the book are balanced.


Since these two forces are of equal magnitude and in opposite directions, they balance each other. The book is said to be at equilibrium. There is no unbalanced force acting upon the book and thus the book maintains its state of motion. When all the forces acting upon an object balance each other, the object will be at equilibrium; it will not accelerate. (Note: diagrams such as the one above are known as free-body diagrams and will be discussed in detail in Lesson 2.)

Consider another example involving balanced forces - a person standing on the floor. There are two forces acting upon the person. The force of gravity exerts a downward force. The floor exerts an upward force.


Since these two forces are of equal magnitude and in opposite directions, they balance each other. The person is at equilibrium. There is no unbalanced force acting upon the person and thus the person maintains its state of motion. (Note: diagrams such as the one above are known as free-body diagrams and will be discussed in detail in Lesson 2.)

## Unbalanced Forces

Now consider a book sliding from left to right across a tabletop. Sometime in the prior history of the book, it may have been given a shove and set in motion from a rest position. Or perhaps it acquired its motion by sliding down an incline from an elevated position. Whatever the case, our focus is not upon the history of the book but rather upon the current situation of a book sliding to the right across a tabletop. The book is in motion and at the moment there is no one pushing it to the right. (Remember: a force is not needed to keep a moving object moving to the right.) The forces acting upon the book are shown below.

## The forces acting on the book are not balanced.



The force of gravity pulling downward and the force of the table pushing upwards on the book are of equal magnitude and opposite directions. These two forces balance each other. Yet there is no force present to balance the force of friction. As the book moves to the right, friction acts to the left to slow the book down. There is an unbalanced force; and as such, the book changes its state of motion. The book is not at equilibrium and subsequently accelerates. Unbalanced forces cause accelerations. In this case, the unbalanced force is directed opposite the book's motion and will cause it to slow down. (Note: diagrams such as the one above are known as free-body diagrams and will be discussed in detail in Lesson 2.)

To determine if the forces acting upon an object are balanced or unbalanced, an analysis must first be conducted to determine what forces are acting upon the object and in what direction. If two individual forces are of equal magnitude and opposite direction, then the forces are said to be balanced. An object is said to be acted upon by an unbalanced force only when there is an individual force that is not being balanced by a force of equal magnitude and in the opposite direction. Such analyses are discussed in Lesson 2 of this unit and applied in Lesson 3.

## Check Your Understanding

Luke Autbeloe drops an approximately 5.0 kg box of shingles (weight $=50.0 \mathrm{~N}$ ) off the roof of his house into the swimming pool below. Upon encountering the pool, the box encounters a 50.0 N upward resistance force (assumed to be constant). Use this description to answer the following questions.

1. Which one of the velocity-time graphs best describes the motion of the box? Support your answer with sound reasoning.




## Explain.

2. Which one of the following dot diagrams best describes the motion of the falling box from the time that they are dropped to the time that they hit the bottom of the pool? The arrows on the diagram represent the point at which the box hits the water. Support your answer with sound reasoning. Explain.

3. Several of Luke's friends were watching the motion of the falling box. Being "physics types", they began discussing the motion and made the following comments. Indicate whether each of the comments is correct or incorrect? Support your answers.
a. Once the box hits the water, the forces are balanced and the box will stop.
b. Upon hitting the water, the box will accelerate upwards because the water applies an upward force.
c. Upon hitting the water, the box will bounce upwards due to the upward force.
4. If the forces acting upon an object are balanced, then the object:
a. must not be moving.
b. must be moving with a constant velocity.
c. must not be accelerating.
d. none of these

## Explain.

## Newton's Laws - Lesson 2 - Force and Its Representation <br> The Meaning of Force

A force is a push or pull upon an object resulting from the object's interaction with another object. Whenever there is an interaction between two objects, there is a force upon each of the objects. When the interaction ceases, the two objects no longer experience the force. Forces only exist as a result of an interaction.

## Contact versus Action-at-a-Distance Forces

For simplicity sake, all forces (interactions) between objects can be placed into two broad categories:

- contact forces, and
- forces resulting from action-at-a-distance

Contact forces are those types of forces that result when the two interacting objects are perceived to be physically contacting each other. Examples of contact forces include frictional forces, tensional forces, normal forces, air resistance forces, and applied forces. These specific forces will be discussed in more detail later in Lesson 2 as well as in other lessons.

Action-at-a-distance forces are those types of forces that result even when the two interacting objects are not in physical contact with each other, yet are able to exert a push or pull despite their physical separation. Examples of action-at-a-distance forces include gravitational forces. For example, the sun and planets exert a gravitational pull on each other despite their large spatial separation. Even when your feet leave the earth and you are no longer in physical contact with the earth, there is a gravitational pull between you and the Earth. Electric forces are action-at-a-distance forces. For example, the protons in the nucleus of an atom and the electrons outside the nucleus experience an electrical pull towards each other despite their small spatial separation. And magnetic forces are action-at-a-distance forces. For example, two magnets can exert a magnetic pull on each other even when separated by a distance of a few centimeters. These specific forces will be discussed in more detail later in Lesson 2 as well as in other lessons.

Examples of contact and action-at-distance forces are listed in the table below.

Contact Forces
Frictional Force
Tension Force
Normal Force
Air Resistance Force
Applied Force
Spring Force

Action-at-a-Distance Forces
Gravitational Force
Electrical Force
Magnetic Force

## The Newton

Force is a quantity that is measured using the standard metric unit known as the Newton. A Newton is abbreviated by an "N." To say "10.0 N" means 10.0 Newton of force. One Newton is the amount of force required to give a $1-\mathrm{kg}$ mass an acceleration of $1 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. Thus, the following unit equivalency can be stated:

$$
1 \text { Newton }=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}
$$

## Force is a Vector Quantity

A force is a vector quantity. As learned in an earlier unit, a vector quantity is a quantity that has both magnitude and direction. To fully describe the force acting upon an object, you must describe both the magnitude (size or numerical value) and the direction. Thus, 10 Newton is not a full description of the force acting upon an object. In contrast, 10 Newton, downward is a complete description of the force acting upon an object; both the magnitude ( 10 Newton) and the direction (downward) are given.

Because a force is a vector that has a direction, it is common to represent forces using diagrams in which a force is represented by an arrow. Such vector diagrams were introduced in an earlier unit and are used throughout the study of physics. The size of the arrow is reflective of the magnitude of the force and the direction of the arrow reveals the direction that the force is acting. (Such diagrams are known as free-body diagrams and are discussed later in this lesson.)
 Furthermore, because forces are vectors, the effect of an individual force upon an object is often canceled by the effect of another force. For example, the effect of a 20-Newton upward force acting upon a book is canceled by the effect of a 20-Newton downward force acting upon the book. In such instances, it is said that the two individual forces balance each other; there would be no unbalanced force acting upon the book.

Other situations could be imagined in which two of the individual vector forces cancel each other ("balance"), yet a third individual force exists that is not balanced by another force. For example, imagine a book sliding across the rough surface of a table from left to right. The downward force of gravity and the upward force of the table supporting the book act in opposite directions and thus balance each other. However, the force of friction acts leftwards,
 and there is no rightward force to balance it. In this case, an unbalanced force acts upon the book to change its state of motion.

The exact details of drawing free-body diagrams are discussed later. For now, the emphasis is upon the fact that a force is a vector quantity that has a direction. The importance of this fact will become clear as we analyze the individual forces acting upon an object later in this lesson.

## Types of Forces

| Type of Force <br> (and Symbol) | Description of Force |
| :--- | :--- |

An applied force is a force that is applied to an object by a person or another object. If a person is pushing a desk across the room, then there

## Applied Force

 is an applied force acting upon the object. The applied force is the force exerted on the desk by the person.$\qquad$
The force of gravity is the force with which the earth, moon, or other massively large object attracts another object towards itself. By
Gravity Force definition, this is the weight of the object. All objects upon earth experience a force of gravity that is directed "downward" towards the center of the earth. The force of gravity on earth is always equal to the
(also known
as Weight) weight of the object as found by the equation:

$$
\text { Fgrav }=\mathbf{m} * \mathbf{g}
$$

$$
\begin{gathered}
\text { where } \mathrm{g}=9.8 \mathrm{~N} / \mathrm{kg} \text { (on Earth) } \\
\text { and } \mathrm{m}=\text { mass (in } \mathrm{kg} \text { ) } \\
\text { (Caution: do not confuse weight with mass.) }
\end{gathered}
$$

$$
\mathrm{F}_{\text {grav }} \quad \text { and } \mathrm{m}=\text { mass (in } \mathrm{kg} \text { ) }
$$

## Normal Force

The normal force is the support force exerted upon an object that is in contact with another stable object. For example, if a book is resting upon a surface, then the surface is exerting an upward force upon the
$\mathrm{F}_{\text {norm }} \quad$ book in order to support the weight of the book. On occasions, a normal force is exerted horizontally between two objects that are in contact with each other. For instance, if a person leans against a wall, the wall pushes horizontally on the person.

Friction Force The friction force is the force exerted by a surface as an object moves across it or makes an effort to move across it. There are at least two types of friction force - sliding and static friction. Thought it is not
$\mathrm{F}_{\text {frict }} \quad$ always the case, the friction force often opposes the motion of an object. For example, if a book slides across the surface of a desk, then the desk exerts a friction force in the opposite direction of its motion. Friction results from the two surfaces being pressed together closely, causing intermolecular attractive forces between molecules of different surfaces. As such, friction depends upon the nature of the two surfaces and upon the degree to which they are pressed together. The maximum amount of friction force that a surface can exert upon an object can be calculated using the formula below:

$$
\mathbf{F}_{\text {frict }}=\boldsymbol{\mu} \bullet \mathbf{F}_{\text {norm }}
$$

The friction force is discussed in more detail later on this page.
\(\left.$$
\begin{array}{ll}\text { Air Resistance Force } & \begin{array}{l}\text { The air resistance is a special type of frictional force that acts upon } \\
\text { objects as they travel through the air. The force of air resistance is often } \\
\text { observed to oppose the motion of an object. This force will frequently }\end{array}
$$ <br>

be neglected due to its negligible magnitude (and due to the fact that it\end{array}\right\}\)| is mathematically difficult to predict its value). It is most noticeable for |
| :--- |
| objects that travel at high speeds (e.g., a skydiver or a downill skier) |
| or for objects with large surface areas. Air resistance will be discussed |
| in more detail in Lesson 3. |

Tension Force The tension force is the force that is transmitted through a string, rope, cable or wire when it is pulled tight by forces acting from opposite ends. The tension force is directed along the length of the wire and $\mathrm{F}_{\text {tens }} \quad$ pulls equally on the objects on the opposite ends of the wire.

The spring force is the force exerted by a compressed or stretched spring upon any object that is attached to it. An object that compresses or stretches a spring is always acted upon by a force that restores the $\mathrm{F}_{\text {spring }} \quad$ object to its rest or equilibrium position. For most springs (specifically, for those that are said to obey "Hooke's Law"), the magnitude of the force is directly proportional to the amount of stretch or compression of the spring.

## Confusion of Mass and Weight

A few further comments should be added about the single force that is a source of much confusion to many students of physics - the force of gravity. As mentioned above, the force of gravity acting upon an object is sometimes referred to as the weight of the object. Many students of physics confuse weight with mass. The mass of an object refers to the amount of matter that is contained by the object; the weight of an object is the force of gravity acting upon that object. Mass is related to how much stuff is there and weight is related to the pull of the Earth (or any other planet) upon that stuff. The mass of an object (measured in kg ) will be the same no matter where in the universe that object is located. Mass is never altered by location, the pull of gravity, speed or even the existence of other forces. For example, a $2-\mathrm{kg}$ object will have a mass of 2 kg whether it is located on Earth, the moon, or Jupiter; its mass will be 2 kg whether it is moving or not (at least for purposes of our study); and its mass will be 2 kg whether it is being pushed upon or not.

On the other hand, the weight of an object (measured in Newton) will vary according to where in the universe the object is. Weight depends upon which planet is exerting the force and the distance the object is from the planet. Weight, being equivalent to the force of gravity, is dependent upon the value of $\mathbf{g}$ - the gravitational field strength. On earth's surface $\mathbf{g}$ is $9.8 \mathrm{~N} / \mathrm{kg}$ (often approximated as $10 \mathrm{~N} / \mathrm{kg}$ ). On the moon's surface, $\boldsymbol{g}$ is $1.7 \mathrm{~N} / \mathrm{kg}$. Go to another planet, and there will be another $\mathbf{g}$ value. Furthermore, the $g$ value is inversely proportional to the distance from the center of the planet. So if we were to measure $\mathbf{g}$ at a distance of 400 km above the earth's surface, then we would find the $\mathbf{g}$ value to be less than $9.8 \mathrm{~N} / \mathrm{kg}$. (The nature of the force of gravity will be discussed in more detail in a later unit of The Physics Classroom.) Always be cautious of the distinction between mass and weight. It is the source of much confusion for many students of physics.

## Flickr Physics Photo

A 1.0-kg mass is suspended from a spring scale in an effort to determine its weight. The scale reads just short of 10.0 N - close enough to call it 9.8 N . Mass refers to how much stuff is present in the object. Weight refers to the force with which gravity pulls upon the object.

## Investigate!

Even on the surface of the Earth, there are local variations in the value of $g$ that have very small effects upon an object's weight. These variations are due to latitude, altitude and the local geological structure of the region. Use the Gravitational Fields widget on the Physics Classroom site to investigate how location affects the value of g. (http://www.physicsclassroom.com/class/newtlaws/Lesson-2/Types-of-Forces)


## Sliding versus Static Friction

As mentioned above, the friction force is the force exerted by a surface as an object moves across it or makes an effort to move across it. For the purpose of our study of physics at The Physics Classroom, there are two types of friction force - static friction and sliding friction. Sliding friction results when an object slides across a surface. As an example, consider pushing a box across a floor. The floor surface offers resistance to the movement of the box. We often say that the floor exerts a friction force upon the box. This is an example of a sliding friction force since it results from the sliding motion of the box. If a car slams on its brakes and skids to a stop (without antilock brakes), there is a sliding friction force exerted upon the car tires by the roadway surface. This friction force is also a sliding friction force because the car is sliding across the road surface. Sliding friction forces can be calculated from knowledge of the coefficient of friction and the normal force exerted upon the object by the surface it is sliding across. The formula is:

## $\mathbf{F}_{\text {frict-sliding }}=\mu_{\text {frict-sliding }} \bullet \mathbf{F}_{\text {norm }}$

The symbol $\mu_{\text {frict-sliding }}$ represents the coefficient of sliding friction between the two surfaces. The coefficient value is dependent primarily upon the nature of the surfaces that are in contact with each other. For most surface combinations, the friction coefficients show little dependence upon other variables such as area of contact, temperature, etc. Values of $\boldsymbol{\mu}_{\text {sliding }}$ have been experimentally determined for a variety of surface combinations and are often tabulated in technical manuals and handbooks. The values of $\mu$ provide a measure of the relative amount of adhesion or attraction of the two surfaces for each other. The more that surface molecules tend to adhere to each other, the greater the coefficient values and the greater the friction force.

Friction forces can also exist when the two surfaces are not sliding across each other. Such friction forces are referred to as static friction. Static friction results when the surfaces of two objects are at rest relative to one another and a force exists on one of the objects to set it into motion relative to the other object. Suppose you were to push with 5-Newton of force on a large box to move it across the floor. The box might remain in place. A static friction force exists between the surfaces of the floor and the box to prevent the box from being set into motion. The static friction force balances the force that you exert on the box such that the stationary box remains at rest. When exerting 5 Newton of applied force on the box, the static friction force has a magnitude of 5 Newton. Suppose that you were to push with 25 Newton of force on the large box and the box were to still remain in place. Static friction now has a magnitude of 25 Newton. Then suppose that you were to increase the force to 26 Newton and the box finally budged from its resting position and was set into motion across the floor. The box-floor surfaces were able to provide up to 25 Newton of static friction force to match your applied force. Yet the two surfaces were not able to provide 26 Newton of static friction force. The amount of static friction resulting from the adhesion of any two surfaces has an upper limit. In this case, the static friction
force spans the range from 0 Newton (if there is no force upon the box) to 25 Newton (if you push on the box with 25 Newton of force). This relationship is often expressed as follows:

$$
\mathbf{F}_{\text {frict-static }} \leq \mu_{\text {frict-static }}{ }^{\bullet} \mathbf{F}_{\text {norm }}
$$

The symbol $\boldsymbol{\mu}_{\text {frict-static }}$ represents the coefficient of static friction between the two surfaces. Like the coefficient of sliding friction, this coefficient is dependent upon the types of surfaces that are attempting to move across each other. In general, values of static friction coefficients are greater than the values of sliding friction coefficients for the same two surfaces. Thus, it typically takes more force to budge an object into motion than it does to maintain the motion once it has been started.

The meaning of each of these forces listed in the table above will have to be thoroughly understood to be successful during this unit. Ultimately, you must be able to read a verbal description of a physical situation and know enough about these forces to recognize their presence (or absence) and to construct a free-body diagram that illustrates their relative magnitude and direction.

## Check Your Understanding

1. Complete the following table showing the relationship between mass and weight.

| Object | Mass (kg) | Weight (N) |
| :---: | :---: | :---: |
| Melon | 1 kg |  |
| Apple |  | 0.98 N |
| Pat Eatladee | 25 kg |  |
| Fred |  | 980 N |

2. Different masses are hung on a spring scale calibrated in Newtons. Find the missing values. The force exerted by gravity on $1 \mathrm{~kg}=9.8 \mathrm{~N}$. Show your work.
a) The force exerted by gravity on $5 \mathrm{~kg}=$ $\qquad$ N.
b) The force exerted by gravity on $\qquad$ $\mathrm{kg}=98 \mathrm{~N}$.
c) The force exerted by gravity on $70 \mathrm{~kg}=$ $\qquad$ N .
3. When a person diets, is their goal to lose mass or to lose weight? Explain.

## Drawing Free-Body Diagrams



Free-body diagrams are diagrams used to show the relative magnitude and direction of all forces acting upon an object in a given situation. A free-body diagram is a special example of the vector diagrams that were discussed in an earlier unit. These diagrams will be used throughout our study of physics. The size of the arrow in a free-body diagram reflects the magnitude of the force. The direction of the arrow shows the direction that the force is acting. Each force arrow in the diagram is labeled to indicate the exact type of force. It is generally customary in a free-body diagram to represent the object by a box and to draw the force arrow from the center of the box outward in the direction that the force is acting. An example of a free-body diagram is shown at the right.

The free-body diagram above depicts four forces acting upon the object. Objects do not necessarily always have four forces acting upon them. There will be cases in which the number of forces depicted by a free-body diagram will be one, two, or three. There is no hard and fast rule about the number of forces that must be drawn in a free-body diagram. The only rule for drawing free-body diagrams is to depict all the forces that exist for that object in the given situation. Thus, to construct free-body diagrams, it is extremely important to know the various types of forces. If given a description of a physical situation, begin by using your understanding of the force types to identify which forces are present. Then determine the direction in which each force is acting. Finally, draw a box and add arrows for each existing force in the appropriate direction; label each force arrow according to its type. If necessary, refer to the list of forces and their description in order to understand the various force types and their appropriate symbols.

## Check for Understanding

Apply the method described in the paragraph above to construct free-body diagrams for the various situations described below. Be sure to include direction, relative magnitude, and indicate type of force. The first one is done for you.

1. A book is at rest on a tabletop. Diagram the forces acting on the book.

2. A girl is suspended motionless from the ceiling by two ropes. Diagram the forces acting on the combination of girl and bar.

3. An egg is free-falling from a nest in a tree. Neglect air resistance. Diagram the forces acting on the egg as it is falling.

4. A flying squirrel is gliding (no wing flaps) from a tree to the ground at constant velocity. Consider air resistance. Diagram the forces acting on the squirrel.

5. A rightward force is applied to a book in order to move it across a desk with a rightward acceleration. Consider frictional forces. Neglect air resistance. Diagram the forces acting on the book.

6. A rightward force is applied to a book in order to move it across a desk at constant velocity. Consider frictional forces. Neglect air resistance. Diagram the forces acting on the book.

7. A college student rests a backpack upon his shoulder. The pack is suspended motionless by one strap from one shoulder. Diagram the vertical forces acting on the backpack.

8. A skydiver is descending with a constant velocity. Consider air resistance. Diagram the forces acting upon the skydiver.

9. A force is applied to the right to drag a sled across loosely packed snow with a rightward acceleration. Neglect air resistance. Diagram the forces acting upon the sled.
10. A football is moving upwards towards its peak after having been booted by the punter. Neglect air resistance. Diagram the forces acting upon the football as it rises upward towards its peak.

11. A car is coasting to the right and slowing down. Neglect air resistance. Diagram the forces acting upon the car.

## Determining the Net Force

If you have been reading through Lessons 1 and 2, then Newton's first law of motion ought to be thoroughly understood.

An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

In the statement of Newton's first law, the unbalanced force refers to that force that does not become completely balanced (or canceled) by the other individual forces. If either all the vertical forces (up and down) do not cancel each other and/or all horizontal forces do not cancel each other, then an unbalanced force exists. The existence of an unbalanced force for a given situation can be quickly realized by looking at the free-body diagram for that situation. Free-body diagrams for three situations are shown below. Note that the actual magnitudes of the individual forces are indicated on the diagram.


In each of the above situations, there is an unbalanced force. It is commonly said that in each situation there is a net force acting upon the object. The net force is the vector sum of all the forces that act upon an object. That is to say, the net force is the sum of all the forces, taking into account the fact that a force is a vector and two forces of equal magnitude and opposite direction will cancel each other out. At this point, the rules for summing vectors (such as force vectors) will be kept relatively simple. Observe the following examples of summing two forces:


Observe in the diagram above that a downward vector will provide a partial or full cancellation of an upward vector. And a leftward vector will provide a partial or full cancellation of a rightward vector. The addition of force vectors can be done in the same manner in order to determine the net force (i.e., the vector sum of all the individual forces). Consider the three situations below in which the net force is determined by summing the individual force vectors that are acting upon the objects.


## A Net Force Causes an Acceleration

As mentioned earlier, a net force (i.e., an unbalanced force) causes an acceleration. In a previous unit, several means of representing accelerated motion (position-time and velocity-time graphs, ticker tape diagrams, velocity-time data, etc.) were discussed. Combine your understanding of acceleration and the newly acquired knowledge that a net force causes an acceleration to determine whether or not a net force exists in the following situations.

Description of Motion
Net Force: Yes or No?
..... . . . . . . . . . चक्षी
. . . . . . . . . . . . . .0e
-
$\qquad$


Description of Motion.


Net Force: Yes or No?



## Check Your Understanding

1. Free-body diagrams for four situations are shown below. For each situation, determine the net force acting upon the object.


Situation C


Situation D

2. Free-body diagrams for four situations are shown below. The net force is known for each situation. However, the magnitudes of a few of the individual forces are not known. Analyze each situation individually and determine the magnitude of the unknown forces.

$F_{\text {net }}=900 \mathrm{~N}$, up
$A=$ $\qquad$ $C=$ $\qquad$ $\mathrm{D}=$ $\qquad$
$\mathrm{E}=$ $\qquad$ $\mathrm{G}=$ $\qquad$
$\mathrm{H}=$ $\qquad$

## Newton's Laws - Lesson 3 - Newton's Second Law of Motion Newton's Second Law

Newton's first law of motion predicts the behavior of objects for which all existing forces are balanced. The first law - sometimes referred to as the law of inertia - states that if the forces acting upon an object are balanced, then the acceleration of that object will be $0 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. Objects at equilibrium (the condition in which all forces balance) will not accelerate. According to Newton, an object will only accelerate if there is a net or unbalanced force acting upon it. The presence of an unbalanced force will accelerate an object - changing its speed, its direction, or both its speed and direction.


Newton's second law of motion pertains to the behavior of objects for which all existing forces are not balanced. The second law states that the acceleration of an object is dependent upon two variables - the net force acting upon the object and the mass of the object. The acceleration of an object depends directly upon the net force acting upon the object, and inversely upon the mass of the object. As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased.


## The BIG Equation

Newton's second law of motion can be formally stated as follows:
The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

This verbal statement can be expressed in equation form as follows:

$$
\mathbf{a}=\mathbf{F}_{\mathrm{net}} / \mathbf{m}
$$

The above equation is often rearranged to a more familiar form as shown below. The net force is equated to the product of the mass times the acceleration.

$$
\mathbf{F}_{\mathrm{net}}=\mathbf{m} \cdot \mathbf{a}
$$

In this entire discussion, the emphasis has been on the net force. The acceleration is directly proportional to the net force; the net force equals mass times acceleration; the acceleration in the same direction as the net force; an acceleration is produced by a net force. The NET FORCE. It is important to remember this distinction. Do not use the value of merely "any 'ole force" in the above equation. It is the net force that is related to acceleration. As discussed in an earlier lesson, the net force is the vector
 sum of all the forces. If all the individual forces acting upon an object are known, then the net force can be determined. If necessary, review this principle by returning to the practice questions in Lesson 2.

Consistent with the above equation, a unit of force is equal to a unit of mass times a unit of acceleration. By substituting standard metric units for force, mass, and acceleration into the above equation, the following unit equivalency can be written.

$$
1 \text { Newton }=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}
$$

The definition of the standard metric unit of force is stated by the above equation. One Newton is defined as the amount of force required to give a $1-\mathrm{kg}$ mass an acceleration of $1 \mathrm{~m} / \mathrm{s} / \mathrm{s}$.

## Your Turn to Practice



The $\mathrm{F}_{\text {net }}=\mathrm{m} \cdot$ a equation is often used in algebraic problem solving. Complete the table below by solving for the unknown quantity.

|  | Net Force <br> $(\mathbf{N})$ | Mass <br> $(\mathbf{k g})$ | Acceleration <br> $(\mathbf{m} / \mathbf{s} / \mathbf{s})$ |
| :---: | :---: | :---: | :---: |
| 1. | 10 | 2 | - |
| 2. | 20 | 2 | - |
| 3. | 20 | 4 | - |
| 4. | - | 2 | 5 |
| 5. | 10 |  | 10 |

## Newton's Second Law as a Guide to Thinking

The numerical information in the table above demonstrates some important qualitative relationships between force, mass, and acceleration. Comparing the values in rows 1 and 2, it can be seen that a doubling of the net force results in a doubling of the acceleration (if mass is held constant). Similarly, comparing the values in rows 2 and 4 demonstrates that a halving of the net force results in a halving of the acceleration (if mass is held constant). Acceleration is directly proportional to net force.

Furthermore, the qualitative relationship between mass and acceleration can be seen by a comparison of the numerical values in the above table. Observe from rows 2 and 3 that a doubling of the mass results in a halving of the acceleration (if force is held constant). And similarly, rows 4 and 5 show that a halving of the mass results in a doubling of the acceleration (if force is held constant). Acceleration is inversely proportional to mass.

The analysis of the table data illustrates that an equation such as $\mathrm{F}_{\mathrm{n}} \mathrm{=}=\mathrm{m}$ * can be a guide to thinking about how a variation in one quantity might affect another quantity. Whatever alteration is made of the net force, the same change will occur with the acceleration. Double, triple or quadruple the net force, and the acceleration will do the same. On the other hand, whatever alteration is made of the mass, the opposite or inverse change will occur with the acceleration. Double, triple or quadruple the mass, and the acceleration will be one-half, one-third or onefourth its original value.

## The Direction of the Net Force and Acceleration

As stated above, the direction of the net force is in the same direction as the acceleration. Thus, if the direction of the acceleration is known, then the direction of the net force is also known. Consider the two oil drop diagrams below for an acceleration of a car. From the diagram, determine the direction of the net force that is acting upon the car. (If necessary, review acceleration from the previous unit.)

## Direction of net force?

## ….......... . . . 0.6

In conclusion, Newton's second law provides the explanation for the behavior of objects upon which the forces do not balance. The law states that unbalanced forces cause objects to accelerate with an acceleration that is directly proportional to the net force and inversely proportional to the mass.

## Rocket Science!

NASA rockets (and others) accelerate upward off the launch pad as they burn a tremendous amount of fuel. As the fuel is burned and exhausted to propel the rocket, the mass of the rocket changes. As such, the same propulsion force can result in increasing acceleration values over time. Use the Rocket Science widget at:
http://www.physicsclassroom.com/class/newtlaws/Lesson-3/Newton-s-Second-Law to explore this effect.
Rocket Science determines the speed of a rocket as a function of time if given the initial mass and the exhaust velocity.

## Check Your Understanding

1. Determine the acceleration that results when a $12-\mathrm{N}$ net force is applied to: a.) a 3-kg object
b.) a $6-\mathrm{kg}$ object.
2. A net force of 15 N is exerted on an encyclopedia to cause it to accelerate at a rate of $5 \mathrm{~m} / \mathrm{s}^{2}$. Determine the mass of the encyclopedia.
3. Suppose that a sled is accelerating at a rate of $2 \mathrm{~m} / \mathrm{s}^{2}$. If the net force is tripled and the mass is doubled, then what is the new acceleration of the sled?
4. Suppose that a sled is accelerating at a rate of $2 \mathrm{~m} / \mathrm{s}^{2}$. If the net force is tripled and the mass is halved, then what is the new acceleration of the sled?

## The Big Misconception

So what's the big deal? Many people have known Newton's first law since eighth grade (or earlier). And if prompted with the first few words, most people could probably recite the law word for word. And what is so terribly difficult about remembering that $\mathrm{F}=\mathrm{ma}$ ? It seems to be a simple algebraic statement for solving story problems. The big deal however is not the ability to recite the first law nor to use the second law to solve problems; but rather the ability to understand their meaning and to believe their implications. While most people know what Newton's laws say, many people do not know what they mean (or simply do not believe what they mean).

## Learning $\neq$ Storing

Cognitive scientists (scientists who study how people learn) have shown that physics students come into physics class with a set of beliefs that they are unwilling (or not easily willing) to discard despite evidence to the contrary. These beliefs about motion (known as misconceptions) hinder further learning. The task of overcoming misconceptions involves becoming aware of the misconceptions, considering alternative conceptions or explanations,
 making a personal evaluation of the two competing ideas and adopting a new conception that is more reasonable than the previously held-misconception. This process involves self-reflection (to ponder your own belief systems), critical thinking (to analyze the reasonableness of two competing ideas), and evaluation (to select the most reasonable and harmonious model that explains the world of motion). Self-reflection, critical thinking, and evaluation. While this process may seem terribly complicated, it is simply a matter of using your noodle (that's your brain).

The most common misconception is one that dates back for ages; it is the idea that sustaining motion requires a continued force. The misconception has already been discussed in a previous lesson, but will now be discussed in more detail. This misconception sticks out its ugly head in a number of different ways and at a number of different times. As your read through the following discussion, give careful attention to your own belief systems. View physics as a system of thinking about the world rather than information that can be dumped into your brain without evaluating its consistency with
 your own belief systems.

Newton's laws declare loudly that a net force (an unbalanced force) causes an acceleration; the acceleration is in the same direction as the net force. To test your own belief system, consider the following question and its answer as seen by clicking the button.

## Are You Infected with the Misconception? Check Your Understanding

Two students are discussing their physics homework prior to class. They are discussing an object that is being acted upon by two individual forces (both in a vertical direction); the free-body diagram for the particular object is shown at the right. During the discussion, Anna Litical suggests to Noah Formula that the object under discussion could be moving. In fact, Anna suggests that if friction and air resistance could be ignored (because of their negligible size),
 the object could be moving in a horizontal direction. According to Anna, an object experiencing forces as described at the right could be experiencing a horizontal motion as described below.

Noah Formula objects, arguing that the object could not have any horizontal motion if there are only vertical forces acting upon it. Noah claims that the object must be at rest, perhaps on a table or floor. After all, says Noah, an object experiencing a balance of forces will be at rest. Who do you agree with? Explain.

Remember last winter when you went sledding down the hill and across the level surface at the local park? (Apologies are extended to those who live in warmer winter climates.)


Imagine a the moment that there was no friction along the level surface from point B to point C and that there was no air resistance to impede your motion. How far would your sled travel? And what would its motion be like? Most students I've talked to quickly answer: the sled would travel forever at constant speed. Without friction or air resistance to slow it down, the sled would continue in motion with the same speed and in the same direction. The forces acting upon the sled from point B to point C would be the normal force (the snow pushes up on the sled) and the gravity force (see diagram at right). These forces are balanced and since
 the sled is already in motion at point B it will continue in motion with the same speed and direction. So, as in the case of the sled and as in the case of the object that Noah and Anna are discussing, an object can be moving to the right even if the only forces acting upon the object are vertical forces. Forces do not cause motion; forces cause accelerations.

## Newton's First Law - Revisited

Newton's first law of motion declares that a force is not needed to keep an object in motion. Slide a book across a table and watch it slide to a rest position. The book in motion on the table top does not come to a rest position because of the absence of a force; rather it is the presence of a force - that force being the force of friction - that brings the book to a rest position. In the absence of a force of friction, the book would continue in motion with the same speed and direction - forever (or at least to the end of the table top)! A force is not required to keep a moving book in motion; and a force is not required to keep a moving sled in motion; and a force is not required to keep any object horizontally moving object in motion. To read more about this misconception, return to an earlier lesson.


## Finding Acceleration

As learned earlier in Lesson 3 (as well as in Lesson 2), the net force is the vector sum of all the individual forces. In Lesson 2, we learned how to determine the net force if the magnitudes of all the individual forces are known. In this lesson, we will learn how to determine the acceleration of an object if the magnitudes of all the individual forces are known. The three major equations that will be useful are the equation for net force $\left(\mathrm{F}_{\text {net }}=\mathrm{m} \bullet \mathrm{a}\right)$, the equation for gravitational force $(\mathrm{Fgrav}=\mathrm{m} \bullet \mathrm{g})$, and the equation for frictional force $\left(\mathrm{Ffrict}=\mu \bullet \mathrm{F}_{\text {norm }}\right)$.

The process of determining the acceleration of an object demands that the mass and the net force are known. If mass ( m ) and net force ( F net) are known, then the acceleration is determined by use of the equation.

$$
\mathbf{a}=\mathbf{F}_{\mathrm{net}} / \mathbf{m}
$$

## Your Turn to Practice



This task involves using the above equations, the given information, and your understanding of Newton's laws to determine the acceleration. To gain a feel for how this method is applied, try the following practice problems. Don't forget to show your work.

## Practice \#1

An applied force of 50 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. Use the diagram to determine the normal force, the net force, the mass, and the acceleration of the object. (Neglect air resistance.) Show your work.


$$
\begin{aligned}
& \mathbf{m}= \\
& \mathbf{a}=
\end{aligned}
$$

$F_{\text {net }}=$

## Practice \#2

An applied force of 20 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. Use the diagram to determine the normal force, the net force, the coefficient of friction $(\mu)$ between the object and the surface, the mass, and the acceleration of the object. (Neglect air resistance.) Show your work.


## Practice \#3

A $5-\mathrm{kg}$ object is sliding to the right and encountering a friction force that slows it down. The coefficient of friction $(\mu)$ between the object and the surface is 0.1 . Determine the force of gravity, the normal force, the force of friction, the net force, and the acceleration. (Neglect air resistance.) Show your work.


$$
\begin{gathered}
\mathrm{m}=5 \mathrm{~kg} \\
\mathrm{ti}= \\
\mathrm{F}_{\mathrm{met}}= \\
\hline
\end{gathered}
$$

A couple more practice problems are provided below. You should make an effort to solve as many problems as you can without the assistance of notes, solutions, teachers, and other students. Commit yourself to individually solving the problems. In the meantime, an important caution is worth mentioning:


Avoid forcing a problem into the form of a previously solved problem. Problems in physics will seldom look the same. Instead of solving problems by rote or by mimicry of a previously solved problem, utilize your conceptual understanding of Newton's laws to work towards solutions to problems. Use your understanding of weight and mass to find the $m$ or the Fgrav in a problem. Use your conceptual understanding of net force (vector sum of all the forces) to find the value of Fnet or the value of an individual force. Do not divorce the solving of physics problems from your understanding of physics concepts. If you are unable to solve physics problems like those above, it is does not necessarily mean that you are having math difficulties. It is likely that you are having a physics concepts difficulty.

## Check Your Understanding

1. Edwardo applies a $4.25-\mathrm{N}$ rightward force to a $0.765-\mathrm{kg}$ book to accelerate it across a tabletop. The coefficient of friction between the book and the tabletop is 0.410 . Determine the acceleration of the book. Show your work.
2. In a physics lab, Kate and Rob use a hanging mass and pulley system to exert a 2.45 N rightward force on a $0.500-\mathrm{kg}$ cart to accelerate it across a low-friction track. If the total resistance force to the motion of the cart is 0.72 N , then what is the cart's acceleration? Show your work.

## Finding Individual Forces

As learned earlier in Lesson 3 (as well as in Lesson 2), the net force is the vector sum of all the individual forces. In Lesson 2, we learned how to determine the net force if the magnitudes of all the individual forces are known. In this lesson, we will learn how to determine the magnitudes of all the individual forces if the mass and acceleration of the object are known. The three major equations that will be useful are the equation for net force ( $\left.\mathrm{F}_{\text {net }}=\mathrm{m} \cdot a\right)$, the equation for gravitational force $\left(\mathrm{F}_{\text {grav }}=\mathrm{m} \bullet \mathrm{g}\right)$, and the equation for frictional force $\left(\mathrm{F}_{\text {frict }}=\mu \bullet \mathrm{F}_{\text {norm }}\right)$.

The process of determining the value of the individual forces acting upon an object involve an application of Newton's second law ( $\mathrm{Fnel}=\mathrm{m} \cdot \mathrm{a}$ ) and an application of the meaning of the net force. If mass (m) and acceleration (a) are known, then the net force (Fnet) can be determined by use of the equation.

$$
\mathbf{F}_{\text {net }}=\mathbf{m} \cdot \mathbf{a}
$$

If the numerical value for the net force and the direction of the net force is known, then the value of all individual forces can be determined. Thus, the task involves using the above equations, the given information, and your understanding of net force to determine the value of individual forces.

## Your Turn to Practice



To gain a feel for how this method is applied, try the following practice problems. The problems progress from easy to more difficult.

## Practice \#1

Free-body diagrams for four situations are shown below. The net force is known for each situation. However, the magnitudes of a few of the individual forces are not known. Analyze each situation individually and determine the magnitude of the unknown forces.


$$
\mathrm{F}_{\text {net }}=0 \mathrm{~N}
$$

$\mathrm{A}=$ $\qquad$ $\mathrm{C}=\square$
$\mathrm{D}=$ $\qquad$

$$
\mathrm{F}=
$$

$\qquad$
$B=$ $\qquad$
$\mathrm{E}=$ $\qquad$

$$
\mathrm{G}=
$$

$\qquad$

$$
\mathrm{H}=
$$

$\qquad$

## Practice \#2

A rightward force is applied to a $6-\mathrm{kg}$ object to move it across a rough surface at constant velocity. The object encounters 15 N of frictional force. Use the diagram to determine the gravitational force, normal force, net force, and applied force. (Neglect air resistance.) Show your work.


## Practice \#3

A rightward force is applied to a $10-\mathrm{kg}$ object to move it across a rough surface at constant velocity. The coefficient of friction between the object and the surface is 0.2 . Use the diagram to determine the gravitational force, normal force, applied force, frictional force, and net force.
(Neglect air resistance.) Show your work.


## Practice \#4

A rightward force is applied to a $5-\mathrm{kg}$ object to move it across a rough surface with a rightward acceleration of $2 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. The coefficient of friction between the object and the surface is 0.1 . Use the diagram to determine the gravitational force, normal force, applied force, frictional force, and net force. (Neglect air resistance.) Show your work.


## Practice \#5

A rightward force of 25 N is applied to a 4-kg object to move it across a rough surface with a rightward acceleration of $2.5 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. Use the diagram to determine the gravitational force, normal force, frictional force, net force, and the coefficient of friction between the object and the surface. (Neglect air resistance.) Show your work.


## Check Your Understanding

1. Lee Mealone is sledding with his friends when he becomes disgruntled by one of his friend's comments. He exerts a rightward force of 9.13 N on his $4.68-\mathrm{kg}$ sled to accelerate it across the snow. If the acceleration of the sled is $0.815 \mathrm{~m} / \mathrm{s} / \mathrm{s}$, then what is the coefficient of friction between the sled and the snow? Draw a force diagram. Show your work.
2. In a Physics lab, Ernesto and Amanda apply a 34.5 N rightward force to a $4.52-\mathrm{kg}$ cart to accelerate it across a horizontal surface at a rate of $1.28 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. Determine the friction force acting upon the cart. Draw a force diagram. Show your work.

## Free Fall and Air Resistance

Previously, it was stated that all objects (regardless of their mass) free fall with the same acceleration $-9.8 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. This particular acceleration value is so important in physics that it has its own peculiar name, the "acceleration of gravity," and its own peculiar symbol, g or ag. But why do all objects free fall at the same rate of acceleration regardless of their mass? Is it because they all weigh the same? ... because they all have the same gravity? ... because the air resistance is the same for each? These questions will be explored in this section of Lesson 3.

In addition to an exploration of free fall, the motion of objects that encounter air resistance will also be analyzed. In particular, two questions will be explored:

- Why do objects that encounter air resistance ultimately reach a terminal velocity?
- In situations in which there is air resistance, why do more massive objects fall faster than less massive objects?
To answer the above questions, Newton's second law of motion ( $\mathrm{F}_{\text {net }}=\mathrm{m} \cdot a$ ) will be applied to analyze the motion of objects that are falling under the sole influence of gravity (free fall) and under the dual influence of gravity and air resistance.


## Free Fall Motion

As learned in an earlier unit, free fall is a special type of motion in which the only force acting upon an object is gravity. Objects that are said to be undergoing free fall, are not encountering a significant force of air resistance; they are falling under the sole influence of gravity. Under such conditions, all objects will fall with the same rate of acceleration, regardless of their mass. But why? Consider the free-falling motion of a $1000-\mathrm{kg}$ baby elephant and a $1-\mathrm{kg}$ overgrown mouse.
$\mathrm{m}=1000 \mathrm{~kg}$


$$
a=\frac{F_{\text {net }}}{m}=\frac{10000 \mathrm{~N}}{1000 \mathrm{~kg}}
$$

$a=10 \mathrm{~m} / \mathrm{s} / \mathrm{s}$
$\mathbf{m}=1 \mathbf{k g}$

$\mathrm{a}=\frac{\mathrm{F}_{\text {met }}}{\mathrm{m}}=\frac{\mathbf{1 0 N}}{1 \mathrm{~kg}}$
$a=10 \mathrm{~m} / \mathrm{s} / \mathrm{s}$

If Newton's second law were applied to their falling motion, and if a free-body diagram were constructed, then it would be seen that the $1000-\mathrm{kg}$ baby elephant would experiences a greater force of gravity. This greater force of gravity would have a direct effect upon the elephant's acceleration; thus, based on force alone, it might be thought that the $1000-\mathrm{kg}$ baby elephant would accelerate faster. But acceleration depends upon two factors: force and mass. The $1000-\mathrm{kg}$ baby elephant obviously has more mass (or inertia). This increased mass has an inverse effect upon the elephant's acceleration. And thus, the direct effect of greater force on the $1000-\mathrm{kg}$ elephant is offset by the inverse effect of the greater mass of the $1000-\mathrm{kg}$ elephant; and so
 each object accelerates at the same rate - approximately $10 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. The ratio of force to mass ( $\mathrm{F}_{\mathrm{nec}} / \mathrm{m}$ ) is the same for the elephant and the mouse under situations involving free fall.

This ratio $\left(\mathrm{F}_{\text {net }} / \mathrm{m}\right)$ is sometimes called the gravitational field strength and is expressed as $9.8 \mathrm{~N} / \mathrm{kg}$ (for a location upon Earth's surface). The gravitational field strength is a property of the location within Earth's gravitational field and not a property of the baby elephant nor the mouse. All objects placed upon Earth's surface will experience this amount of force ( 9.8 N ) upon every 1 kilogram of mass within the object. Being a property of the location within Earth's gravitational field and not a property of the free falling object itself, all objects on Earth's surface will experience this amount of force per mass. As such, all objects free fall at the same rate regardless of their mass. Because the $9.8 \mathrm{~N} / \mathrm{kg}$ gravitational field at Earth's surface causes a $9.8 \mathrm{~m} / \mathrm{s} / \mathrm{s}$ acceleration of any object placed there, we often call this ratio the acceleration of gravity. (Gravitational forces will be discussed in greater detail in a later unit of The Physics Classroom tutorial.)


## Look It Up!

The value of the gravitational field strength $(\mathbf{g})$ is different in different gravitational environments. Use the Value of $\mathbf{g}$ widget at: http://www.physicsclassroom.com/class/newtlaws/Lesson-3/Free-Fall-and-Air-Resistance to look up the the gravitational field strength on other planets.

## Investigate!

Even on the surface of the Earth, there are local variations in the value of g. These variations are due to latitude (the Earth isn't a perfect sphere; it buldges in the middle), altitude and the local geological structure of the region. Use the Gravitational Fields widget at: http://www.physicsclassroom.com/class/newtlaws/Lesson-3/Free-Fall-and-Air-Resistance

## Falling with Air Resistance

As an object falls through air, it usually encounters some degree of air resistance. Air resistance is the result of collisions of the object's leading surface with air molecules. The actual amount of air resistance encountered by the object is dependent upon a variety of factors. To keep the topic simple, it can be said that the two most common factors that have a direct effect upon the amount of air resistance are the speed of the object and the cross-sectional area of the object. Increased speeds result in an
 increased amount of air resistance. Increased cross-sectional areas result in an increased amount of air resistance.


Why does an object that encounters air resistance eventually reach a terminal velocity? To answer this questions, Newton's second law can be applied to the motion of a falling skydiver.

In the diagrams below, free-body diagrams showing the forces acting upon an $85-\mathrm{kg}$ skydiver (equipment included) are shown. For each case, use the diagrams to determine the net force and acceleration of the skydiver at each instant in time. Show your work.


The previous diagrams illustrate a key principle. As an object falls, it picks up speed. The increase in speed leads to an increase in the amount of air resistance. Eventually, the force of air resistance becomes large enough to balances the force of gravity. At this instant in time, the net force is 0 Newton; the object will stop accelerating. The object is said to have reached a terminal velocity. The change in velocity terminates as a result of the balance of forces. The velocity at which this happens is called the terminal velocity.

> Animation

In situations in which there is air resistance, more massive objects fall faster than less massive objects. But why? To answer the why question, it is necessary to consider the free-body diagrams for objects of different mass. Consider the falling motion of two skydivers: one with a mass of 100 kg (skydiver plus parachute) and the other with a mass of 150 kg (skydiver plus parachute). The free-body diagrams are shown below for the instant in time in which they have reached terminal velocity.


As learned above, the amount of air resistance depends upon the speed of the object. A falling object will continue to accelerate to higher speeds until they encounter an amount of air resistance that is equal to their weight. Since the $150-\mathrm{kg}$ skydiver weighs more (experiences a greater force of gravity), it will accelerate to higher speeds before reaching a terminal velocity. Thus, more massive objects fall faster than less massive objects because they are acted upon by a larger force of gravity; for this reason, they accelerate to higher speeds until the air resistance force equals the gravity force.

## Investigate!

The amount of air resistance an object experiences depends on its speed, its cross-sectional area, its shape and the density of the air. Air densities vary with altitude, temperature and humidity. Nonetheless, $1.29 \mathrm{~kg} / \mathrm{m}^{3}$ is a very reasonable value. The shape of an object affects the drag coefficient $(\mathbf{C d})$. Values for various shapes can be found here. Try the What a Drag! widget at: http://www.physicsclassroom.com/class/newtlaws/Lesson-3/Free-Fall-and-Air-Resistance
Newton's Laws - Lesson 3 - Newton's Second Law of Motion

## Double Trouble (a.k.a., Two Body Problems)

Our study thus far has been restricted to the analysis of single objects moving under the influence of Newton's laws. But what happens if there are two objects connected together in one way or another? For instance, there could be a tow truck hauling a car down a highway. How is such an analysis conducted? How is the acceleration of the tow truck and the car determined? What about the force acting between the tow truck and the car? In this part of Lesson 3, we will make an attempt to analyze such situations. We will find that the analysis is conducted in the same general manner as when there is one object - through the use of free-body diagrams and Newton's laws.

## The Basic Approach

Situations involving two objects are often referred to as two-body situations. When appearing as physics problems, two-body problems are characterized by a set of two unknown quantities. Most commonly (though not always the case), the two unknowns are the acceleration of the two objects and the force transmitted between the two objects. Two body-problems can typically be approached using one of two basic approaches. One approach involves a combination of a system analysis and an individual body analysis. In the system analysis, the two objects are considered to be a single object moving (or accelerating) together as a whole. The mass of the system is the sum of the mass of the two individual objects. If acceleration is involved, the acceleration of the system is the same as that of the individual objects. A system analysis is usually performed to determine the acceleration of the system. The system analysis is combined with an individual object analysis. In the individual object analysis, either one of the two objects is isolated and considered as a separate, independent object. A free-body diagram is constructed and the individual forces acting upon the object are identified and calculated. An individual object analysis is usually performed in order to determine the value of any force which acts between the two objects - for example, contact forces or tension forces.

The dual combination of a system analysis and an individual object analysis is one of two approaches that are typically used to analyze two-body problems. A second approach involves the use of two separate individual object analyses. In such an approach, free-body diagrams are constructed independently for each object and Newton's second law is used to relate the individual force values to the mass and acceleration. Each individual object analysis generates an equation with an unknown. The result is a system of two equations with two unknowns. The system of equations is solved in order to determine the unknown values.

As a first example of the two approaches to solving two-body problems, consider the following example problem.

## Example Problem 1:

A $5.0-\mathrm{kg}$ and a $10.0-\mathrm{kg}$ box are touching each other. A $45.0-\mathrm{N}$ horizontal force is applied to the $5.0-\mathrm{kg}$ box in order to accelerate both boxes across the floor.
 Ignore friction forces and determine the acceleration of the boxes and the force acting between the boxes.

The first approach to this problem involves the dual combination of a system analysis and an individual object analysis. As mentioned, the system analysis is used to determine the acceleration and the individual object analysis is used to determine the forces acting between the objects. In the system analysis, the two objects are considered to be a single object. The dividing line that separates the objects is ignored. The mass of the system of two objects is 15.0 kg . The free-body diagram for the system is shown at the right. There are three forces acting upon the system - the gravity force (the Earth pulls down on the 15.0 kg of mass), the normal force (the floor pushes up on the system to support its weight),
 and the applied force (the hand is pushing on the back part of the system). The force acting between the $5.0-\mathrm{kg}$ box and the $10.0-\mathrm{kg}$ box is not considered in the system analysis since it is an internal force. Just as the forces holding atoms together within an object are not included in a free-body diagram, so the forces holding together the parts of a system are ignored. These are considered internal forces; only external forces are considered when drawing free-body diagrams. The magnitude of the force of gravity is $\mathbf{m} \bullet g$ or 147 N . The magnitude of the normal force is also 147 N since it must support the weight ( 147 N ) of the system. The applied force is stated to be 45.0 N . Newton's second law ( $\mathbf{a}=\mathbf{F}_{\text {net }} / \mathbf{m}$ ) can be used to determine the acceleration. Using 45.0 N for $\mathbf{F}_{\text {net }}$ and 15.0 kg for $\mathbf{m}$, the acceleration is $3.0 \mathrm{~m} / \mathrm{s}^{2}$.

Now that the acceleration has been determined, an individual object analysis can be performed on either object in order to determine the force acting between them. It does not matter which object is chosen; the result will be the same in either case. Here the individual object analysis is conducted on the 10.0 kg object (only because there is one less force acting on it). The free-body diagram for the $10.0-\mathrm{kg}$ object is shown at the right. There are only three forces acting upon
 it - the force of gravity on the $10.0-\mathrm{kg}$, the support force (from the floor pushing upward) and the rightward contact force ( $\mathbf{F}_{\text {contact }}$ ). As the $5.0-\mathrm{kg}$ object accelerates to the right, it will be pushing rightward upon the $10.0-\mathrm{kg}$ object; this is known as a contact force (or a normal force or an applied force or ...). The vertical forces balance each other since there is no vertical acceleration. The only unbalanced force on the $10.0-\mathrm{kg}$ object is the Fcontact. This force is the net force and is equal to $\mathbf{m} \cdot \mathbf{a}$ where $\mathbf{m}$ is equal to 10.0 kg (since this analysis is for the $10.0-\mathrm{kg}$ object) and a was already determined to be $3.0 \mathrm{~m} / \mathrm{s}^{2}$. The net force is equal to 30.0 N . This net force is the force of
the $5.0-\mathrm{kg}$ object pushing the $10.0-\mathrm{kg}$ object to the right; it has a magnitude of 30.0 N . So the answers to the two unknowns for this problem are $3.0 \mathrm{~m} / \mathrm{s}^{2}$ and 30.0 N .

Now we will consider the solution to this same problem using the second approach - the use of two individual object analyses. In the process of this second approach, we will ignore the fact that we know what the answers are and presume that we are solving the problem for the first time. In this approach, two separate free-body diagram analyses are performed. The diagrams below show the free-body diagrams for the two objects.

## Free-Body Diagrams for Individual Objects



Note that there are four forces on the $5.0-\mathrm{kg}$ object at the rear. The two vertical forces $-\mathrm{F}_{\text {grav }}$ and $\mathrm{F}_{\text {norm }}$ - are obvious forces. The $45.0-\mathrm{N}$ applied force ( $\mathrm{F}_{\text {app }}$ ) is the result of the hand pushing on the rear object as described in the problem statement and depicted in the diagram. The leftward contact force on the $5.0-\mathrm{kg}$ object is the force of the $10.0-\mathrm{kg}$ object pushing leftward on the $5.0-$ kg object. As an attempt is made to push the rear object ( $5.0-\mathrm{kg}$ object) forward, the front object ( $10.0-\mathrm{kg}$ object) pushes back upon it. This force is equal to and opposite of the rear object pushing forward on the front object. This force is simply labeled as $\mathrm{F}_{\text {contact }}$ for both of the freebody diagrams. In the free-body diagram for the $10.0-\mathrm{kg}$ object, there are only three forces. Once more, the two vertical forces $-\mathrm{F}_{\text {grav }}$ and $\mathrm{F}_{\text {norm }}$ - are obvious forces. The horizontal force is simply the $5.0-\mathrm{kg}$ object pushing the $10.0-\mathrm{kg}$ object forward. The 45.0 N applied force is not exerted upon this $10.0-\mathrm{kg}$ object; it is exerted on the $5.0-\mathrm{kg}$ object and has already been considered in the previous free-body diagram.

Now the goal of this approach is to generate system of two equations capable of solving for the two unknown values. Using $\mathrm{F}_{\text {net }}=\mathrm{m} \cdot \mathrm{a}$ with the free-body diagram for the $5.0-\mathrm{kg}$ object will yield the Equation 1 below:

$$
45.0-\mathrm{F}_{\text {contact }}=5.0 \bullet \mathrm{a}
$$

Using $\mathrm{F}_{\text {net }}=\mathrm{m} \cdot \mathrm{a}$ with the free-body diagram for the $10.0-\mathrm{kg}$ object will yield the
$\longleftarrow$ Equation 1 Equation 2 below:

$$
\mathrm{F}_{\text {contact }}=10.0 \cdot \mathrm{a}
$$

(Note that the units have been dropped from Equations 1 and 2 in order to clean the $\longleftarrow$ Equation 2 equations up.) If the expression $10.0 \cdot \mathrm{a}$ is substituted into Equation 1 for $\mathrm{F}_{\text {contact }}$, then Equation 1 becomes reduced to a single equation with a single unknown. The equation becomes

$$
45.0-10.0 \cdot a=5.0 \cdot \mathrm{a}
$$

A couple of steps of algebra lead to an acceleration value of $\mathbf{3 . 0} \mathbf{~ m} / \mathrm{s}^{2}$. This value of a can be substituted back into Equation 2 in order to determine the contact force:

$$
\begin{gathered}
\mathrm{F}_{\text {contact }}=10.0 \bullet \mathrm{a}=10.0 \cdot 3.0 \\
\mathbf{F}_{\text {contact }}=\mathbf{3 0 . 0} \mathbf{~ N}
\end{gathered}
$$

As can be seen, using the second approach to solve two body problems yields the same two answers for the two unknowns. Now we will try the same two approaches on a very similar problem that includes a friction force.

## Example Problem 2:

A $5.0-\mathrm{kg}$ and a $10.0-\mathrm{kg}$ box are touching each other. A $45.0-\mathrm{N}$ horizontal force is applied to the $5.0-\mathrm{kg}$ box in order to accelerate both boxes across the floor.
 The coefficient of kinetic friction is 0.200 . Determine the acceleration and the contact force.

Our first solution to this problem will involve the dual combination of a system analysis and an individual object analysis. As you likely noticed, Example Problem 2 is similar to Example Problem 1 with the exception that the surface is not frictionless in Example Problem 2. So when conducting the system analysis in this second example, the friction on the $15-\mathrm{kg}$ system must be considered. So the free-body diagram for the system
 now includes four forces - the same three as in Example Problem 1 plus a leftward force of friction. The force of friction on the system can be calculated as $\mu \bullet \mathrm{F}_{\text {norm }}$ where $\mathrm{F}_{\text {norm }}$ is the normal force experienced by the system. The $\mathrm{F}_{\text {norm }}$ of the system is equal to the force of gravity acting upon the $15.0-\mathrm{kg}$ system; this value is 147 N . So

$$
\mathrm{F}_{\text {frict }}=\mu \bullet \mathrm{F}_{\text {norm }}=(0.200) \bullet(147 \mathrm{~N})=29.4 \mathrm{~N}
$$

The vertical forces balance each other - consistent with the fact that there is no vertical acceleration. The horizontal forces do not balance each other. The net force can be determined as the vector sum of $\mathrm{F}_{\text {app }}$ and $\mathrm{F}_{\text {frict. }}$. That is, $\mathrm{F}_{\text {net }}=45.0 \mathrm{~N}$, right +29.4 N , left; these add to 15.6 N , right. The acceleration can now be calculated using Newton's second law.

$$
\mathrm{a}=\mathrm{F}_{\text {net }} / \mathrm{m}=(15.6 \mathrm{~N} / 15.0 \mathrm{~kg})=\mathbf{1 . 0 4} \mathbf{~ m} / \mathbf{s}^{2}
$$

Now that the system analysis has been used to determine the acceleration, an individual object analysis can be performed on either object in order to determine the force acting between them. Once more, it does not matter which object is chosen; the result would be the same in either case. The
 $10.0-\mathrm{kg}$ object is chosen for the individual object analysis because there is one less force acting upon it; this makes the solution easier. There are four forces acting upon the $10.0-\mathrm{kg}$ object. The two vertical forces are obvious - the force of gravity $(98.0 \mathrm{~N})$ and the normal force (equal to the force of gravity). The horizontal forces are the friction force to the left and the
force of the $5.0-\mathrm{kg}$ object pushing the $10.0-\mathrm{kg}$ object forward; this is labeled as $\mathrm{F}_{\text {contact }}$ on the free-body diagram. The net force - vector sum of all the forces - can always be found by adding the forces in the direction of the acceleration and subtracting those that are in the opposite direction. This $\mathrm{F}_{\text {net }}$ is equal to $\mathrm{F}_{\text {contact }}-\mathrm{F}_{\text {frict }}$. Applying Newton's second law to this object yields the equation:

$$
\mathrm{F}_{\text {contact }}-\mathrm{F}_{\text {frict }}=(10.0 \mathrm{~kg}) \cdot\left(1.04 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

The friction force on this $10.0-\mathrm{kg}$ object is not the same as the friction force on the system (since the system was weightier). The $\mathrm{F}_{\text {frict }}$ value can be computed as $\mu \bullet \mathrm{F}_{\text {norm }}$ where $\mathrm{F}_{\text {norm }}$ is the normal force experienced by the $10.0-\mathrm{kg}$ object. The $\mathrm{F}_{\text {norm }}$ of the $10.0-\mathrm{kg}$ is equal to the force of gravity acting upon the $10.0-\mathrm{kg}$ object; this value is 98.0 N . So

$$
\mathrm{F}_{\text {frict }}=\mu \cdot \mathrm{F}_{\text {norm }}=(0.200) \bullet(98.0 \mathrm{~N})=19.6 \mathrm{~N}
$$

So now the value of 19.6 N can be substituted into the above equation and $\mathrm{F}_{\text {contact }}$ can be calculated:

$$
\begin{gathered}
\mathrm{F}_{\text {contact }}-19.6 \mathrm{~N}=(10.0 \mathrm{~kg}) \cdot\left(1.04 \mathrm{~m} / \mathrm{s}^{2}\right) \\
\mathrm{F}_{\text {contact }}=(10.0 \mathrm{~kg}) \cdot\left(1.04 \mathrm{~m} / \mathrm{s}^{2}\right)+19.6 \mathrm{~N} \\
\mathbf{F}_{\text {contact }}=\mathbf{3 0 . 0} \mathbf{~ N}
\end{gathered}
$$

So using the dual combination of the system analysis and individual body analysis allows us to determine the two unknown values $-1.04 \mathrm{~m} / \mathrm{s}^{2}$ for the acceleration and 30.0 N for the $F_{\text {contact }}$.

Now we will see how two individual object analyses can be combined to generate a system of two equations capable of solving for the two unknowns. Once more we will start the analysis by presuming that we are solving the problem for the first time and do not know the acceleration nor the contact force. The free-body diagrams for the individual objects are shown below.

## Free-Body Diagrams for Individual Objects



There are now five forces on the $5.0-\mathrm{kg}$ object at the rear. The two vertical forces $-\mathrm{F}_{\text {grav }}$ and $\mathrm{F}_{\text {norm }}$ - are obvious forces. The $45.0-\mathrm{N}$ applied force ( $\mathrm{F}_{\text {app }}$ ) is the result of the hand pushing on the rear object. The leftward contact force on the $5.0-\mathrm{kg}$ object is the force of the $10.0-\mathrm{kg}$ object pushing leftward on the $5.0-\mathrm{kg}$ object. Its value is the same as the contact force that is exerted on the front $10.0-\mathrm{kg}$ object by the rear $5.0-\mathrm{kg}$ object. This force is simply labeled as $\mathrm{F}_{\text {contact }}$ for both of the free-body diagrams. Finally, the leftward friction force is the result of friction with the floor over which the $5.0-\mathrm{kg}$ object moves. In the free-body diagram for the $10.0-\mathrm{kg}$ object, there are now four forces. The two vertical forces - $\mathrm{F}_{\text {grav }}$ and $\mathrm{F}_{\text {norm }}$ - are obvious. The rightward contact force ( $\mathrm{F}_{\text {contact }}$ ) is simply the $5.0-\mathrm{kg}$ object pushing the $10.0-\mathrm{kg}$ object forward. And the leftward friction
force is the result of friction with the floor. Once more, the 45.0 N applied force is not exerted upon this $10.0-\mathrm{kg}$ object; it is exerted on the $5.0-\mathrm{kg}$ object and has already been considered in the previous free-body diagram. The friction force for each object can be determined as $\mu \bullet$ Fnorm where $\mathbf{F}_{\text {norm }}$ is the normal force experienced by the individual objects. Each object experiences a normal force equal to its weight (since vertical forces must balance). So the friction forces for the $5.0-\mathrm{kg}$ object ( 49.0 N weight) and $10.0-\mathrm{kg}$ object ( 98.0 N weight) are $0.200 \cdot 49.0 \mathrm{~N}$ and $0.200 \cdot 98.0 \mathrm{~N}$, respectively.

Using these $\mathrm{F}_{\text {frict }}$ values and Newton's second law, a system of two equations capable of solving for the two unknown values can be written. Using $\mathrm{F}_{\text {net }}=\mathrm{m} \bullet a$ with the free-body diagram for the $5.0-\mathrm{kg}$ object will yield Equation 3 below:

$$
45.0-\mathbf{F}_{\text {contact }}-9.8=5.0 \bullet \mathrm{a}
$$

Using $\mathrm{F}_{\text {net }}=\mathrm{m} \cdot$ a with the free-body diagram for the $10.0-\mathrm{kg}$ object will yield the
Equation 3 Equation 4 below:

$$
\mathbf{F}_{\text {contact }}-19.6=10.0 \cdot \mathrm{a}
$$

(Note that the units have been dropped from Equations 3 and 4 in order to clean the $\longleftarrow$ Equation 4 equations up.) From Equation 4, $\mathbf{F}_{\text {contact }}=10.0 \bullet a+19.6$. Substituting this expression for $\mathbf{F}_{\text {contact }}$ into Equation 3 and performing proper algebraic manipulations yields the acceleration value:

$$
\begin{gathered}
45.0-(10.0 \bullet \mathrm{a}+19.6)-9.8=5.0 \bullet \mathrm{a} \\
45.0-19.6-9.8=15.0 \bullet \mathrm{a} \\
15.6=15.0 \cdot \mathrm{a} \\
\mathrm{a}=(15.6 / 15.0)=\mathbf{1 . 0 4} \mathbf{~ m} / \mathrm{s}^{2}
\end{gathered}
$$

This acceleration value can be substituted back into the expression for $\mathbf{F}_{\text {contact }}$ in order to determine the contact force:

$$
\begin{gathered}
\mathbf{F}_{\text {contact }}=10.0 \bullet \mathbf{a}+19.6=10.0 \bullet(1.04)+19.6 \\
\mathbf{F}_{\text {contact }}=\mathbf{3 0 . 0} \mathbf{~ N}
\end{gathered}
$$

Again we find that the second approach of using two individual object analyses yields the same set of answers for the two unknowns. The final example problem will involve a vertical motion. The approaches will remain the same.

## Example Problem 3:

A man enters an elevator holding two boxes - one on top of the other. The top box has a mass of 6.0 kg and the bottom box has a mass of 8.0 kg . The man sets the two boxes on a metric scale sitting on the floor.
 When accelerating upward from rest, the man observes that the scale reads a value of 166 N ; this is the upward force upon the bottom box. Determine the acceleration of the elevator (and boxes) and determine the forces acting between the boxes.

Both approaches will be used to solve this problem. The first approach involves the dual combination of a system analysis and an individual object analysis. For the system analysis, the two boxes are considered to be a single system with a mass of 14.0 kg . There are two forces acting upon this system - the force of gravity and the normal force. The free-body diagram is shown at the right. The force of gravity is
 calculated in the usual manner using 14.0 kg as the mass.

$$
\mathrm{F}_{\text {grav }}=\mathrm{m} \bullet \mathrm{~g}=14.0 \mathrm{~kg} \bullet 9.8 \mathrm{~N} / \mathrm{kg}=137.2 \mathrm{~N}
$$

Since there is a vertical acceleration, the vertical forces will not be balanced; the $\mathrm{F}_{\text {grav }}$ is not equal to the $\mathrm{F}_{\text {norm }}$ value. The normal force is provided in the problem statement. This $166-\mathrm{N}$ normal force is the upward force exerted upon the bottom box; it serves as the force on the system since the bottom box is part of the system. The net force is the vector sum of these two forces. So:

$$
\mathrm{F}_{\text {net }}=166 \mathrm{~N}, \mathrm{up}+137.2 \mathrm{~N}, \text { down }=28.8 \mathrm{~N}, \mathrm{up}
$$

The acceleration can be calculated using Newton's second law:

$$
\mathbf{a}=\mathbf{F}_{\text {net }} / \mathbf{m}=28.8 \mathrm{~N} / 14.0 \mathrm{~kg}=2.0571 \mathrm{~m} / \mathrm{s}^{2}=\sim 2.1 \mathrm{~m} / \mathrm{s}^{2}
$$

Now that the system analysis has been used to determine the acceleration, an individual object analysis can be performed on either box in order to determine the force acting between them. As in the previous problems, it does not matter which box is chosen; the result will be the same in either
 case. The top box is used in this analysis since it encounters one less force. The free-body diagram is shown at the right. The force of gravity on the top box is $\mathrm{m} \bullet \mathrm{g}$ where $\mathbf{m}=6.0 \mathrm{~kg}$. The force of gravity is 58.8 N . The upward force is not known but can be calculated if the $\mathbf{F}_{\text {net }}=\mathrm{m} \cdot$ a equation is applied to the free-body diagram. Since the acceleration is upward, the Fnet side of the equation would be equal to the force in the direction of the acceleration $\left(\mathbf{F}_{\text {contact }}\right)$ minus the force that opposes it ( $\mathrm{F}_{\text {grav }}$ ). So:

$$
\mathbf{F}_{\text {contact }}-58.8 \mathrm{~N}=(6.0 \mathrm{~kg}) \bullet(2.0571 \mathrm{~m} / \mathrm{s} 2)
$$

(Notice that the unrounded value of acceleration is used here; rounding will occur when the final answer is determined.) Solving for $\mathbf{F}_{\text {contact }}$ yields 71.14 N . This figure can be rounded to two significant digits - $\mathbf{7 1} \mathbf{N}$. So the dual combination of the system analysis and the individual body analysis leads to an acceleration of $2.1 \mathrm{~m} / \mathrm{s}^{2}$ and a contact force of 71 N .

Now the second problem-solving approach will be used to solve the same problem. In this solution, two individual object analyses will be combined to generate a system of two equations capable of solving for the two unknowns. We will start this analysis by presuming that we are solving the problem for the first time and do not know the acceleration nor the contact force. The free-body diagrams for the individual objects are shown below.

## Free-Body Diagrams for Individual Objects



Note that the $\mathrm{F}_{\text {grav }}$ values for the two boxes have been included on the diagram. These were calculated using $\mathrm{F}_{\text {grav }}=\mathrm{m} \bullet \mathrm{g}$ where $\mathrm{m}=6.0 \mathrm{~kg}$ for the top box and $\mathrm{m}=8.0 \mathrm{~kg}$ for the bottom box. The contact force ( $\mathbf{F}_{\text {contact }}$ ) on the top box is upward since the bottom box is pushing it upward as the system of two objects accelerates upward. The contact force $\left(\mathbf{F}_{\text {contact }}\right)$ on the bottom box is downward since the top box pushes downward on the bottom box as the acceleration occurs. These two contact forces are equal to one another since they result from a mutual interaction between the two boxes. The third force on the bottom box is the force of the scale pushing upward on it with 166 N of force; this value was given in the problem statement.

Applying Newton's second law to these two free-body diagrams leads to Equation 5 (for the 6.0kg box) and Equation 6 (for the $8.0-\mathrm{kg}$ box).

$$
\begin{array}{ll}
\mathrm{F}_{\text {contact }}-58.8=6.0 \cdot \mathrm{a} & \longmapsto \text { Equation 5 } \\
166-\mathrm{F}_{\text {contact }}-78.4=8.0 \cdot \mathrm{a} & \longleftarrow \text { Equation 6 }
\end{array}
$$

Now that a system of two equations has been developed, algebra can be used to solve for the two unknowns. Equation 5 can be used to write an expression for the contact force ( $\mathrm{F}_{\text {contact }}$ ) in terms of the acceleration (a). $\quad \mathrm{F}_{\text {contact }}=6.0 \cdot \mathrm{a}+58.8$
This expression for $\mathrm{F}_{\text {contact }}$ can then be substituted into equation 6. Equation 6 then becomes

$$
166-(6.0 \cdot a+58.8)-78.4=8.0 \cdot \mathrm{a}
$$

The following algebraic steps are performed on the above equation to solve for acceleration.

$$
\begin{gathered}
166-6.0 \cdot \mathrm{a}-58.8-78.4=8.0 \cdot \mathrm{a} \\
166-58.8-78.4=8.0 \cdot \mathrm{a}+6.0 \cdot \mathrm{a} \\
28.8=14.0 \mathrm{a} \\
\mathrm{a}=2.0571 \mathrm{~m} / \mathrm{s}^{2}=\sim 2.1 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$

Now the value for acceleration (a) can be substituted back into the expression for $\mathrm{F}_{\text {contact }}\left(\mathrm{F}_{\text {contact }}\right.$ $=6.0 \cdot \mathrm{a}+58.8)$ to solve for $\mathrm{F}_{\text {contact. }}$. The contact force is $71.14 \mathrm{~N}(\sim 71 \mathrm{~N})$.

It should be noted that the second approach to this problem yields the same numerical answers as the first approach. Students are encouraged to use the approach that they are most comfortable with.

## Check Your Understanding

Solve the following problems using either method. Draw a force diagram. Show your work.

1. A truck hauls a car cross-country. The truck's mass is $4.00 \times 10^{3} \mathrm{~kg}$ and the car's mass is $1.60 \times 10^{3} \mathrm{~kg}$. If the force of propulsion resulting from the truck's turning wheels is $2.50 \times 10^{4} \mathrm{~N}$, then determine the acceleration of the car (or the truck) and the force at which the truck pulls upon the car. Assume negligible air resistance forces.
2. A $7.00-\mathrm{kg}$ box is attached to a $3.00-\mathrm{kg}$ box by rope 1 . The $7.00-\mathrm{kg}$ box is pulled by rope 2 with a force of 25.0 N .
Determine the acceleration of the boxes and the tension in rope
3. The coefficient of friction between the ground and the boxes
 is 0.120 .
4. A tractor is being used to pull two large logs across a field. A chain connects the logs to each other; the front $\log$ is connected to the tractor by a separate chain. The mass of the front $\log$ is 180 kg . The mass of the back $\log$ is 220 kg . The coefficient of friction between the logs and the field is approximately 0.45 . The tension in the chain connecting the tractor to the front $\log$ is 1850 N. Determine the acceleration of the logs and the tension in the chain that connects the two logs.
5. Two boxes are held together by a strong wire and attached to the ceiling of an elevator by a second wire (see diagram). The mass of the top box is 14.2 kg ; the mass of the bottom box is 10.4 kg . The elevator accelerates upwards at $2.84 \mathrm{~m} / \mathrm{s}^{2}$. (Assume the wire is relatively massless.)
a.) Find the tension in the top wire (connecting points A and B).
b.) Find the tension in the bottom wire (connecting points C and D ).


## Newton's Laws - Lesson 4 - Newton's Third Law of Motion

## Newton's Third Law

A force is a push or a pull that acts upon an object as a results of its interaction with another object. Forces result from interactions! As discussed in Lesson 2, some forces result from contact interactions (normal, frictional, tensional, and applied forces are examples of contact forces) and other forces are the result of action-at-a-distance interactions (gravitational, electrical, and magnetic forces). According to Newton, whenever objects A and B interact with each other, they exert forces upon each other. When you sit in your chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body. There are two forces resulting from this interaction - a force on the chair and a force on your body. These two forces are called action and reaction forces and are the subject of Newton's third law of motion. Formally stated, Newton's third law is:

For every action, there is an equal and opposite reaction.
The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs.


## Examples of Interaction Force Pairs

A variety of action-reaction force pairs are evident in nature. Consider the propulsion of a fish through the water. A fish uses its fins to push water backwards. But a push on the water will only serve to accelerate the water. Since forces result from mutual interactions, the water must also be pushing the fish forwards, propelling the fish through the water. The size of the force on the water equals the size of the force on the fish; the direction of the force on the water (backwards) is opposite the direction of the force on the fish (forwards). For every action, there is an equal (in size) and opposite (in direction) reaction force. Action-reaction force pairs make it possible for fish to swim.

Consider the flying motion of birds. A bird flies by use of its wings. The wings of a bird push air downwards. Since forces result from mutual interactions, the air must also be pushing the bird upwards. The size of the force on the air equals the size of the force on the bird; the direction of the force on the air (downwards) is opposite the direction of the force on the bird (upwards). For every action, there is an equal (in size) and opposite (in direction) reaction. Action-reaction force pairs make it possible for birds to fly.


Consider the motion of a car on the way to school. A car is equipped with wheels that spin. As the wheels spin, they grip the road and push the road backwards. Since forces result from mutual interactions, the road must also be pushing the wheels forward. The size of the force on the road equals the size of the force on the wheels (or car); the direction of the force on the road (backwards) is opposite the direction of the force on the wheels (forwards). For every action, there is an equal (in size) and opposite (in direction) reaction. Action-reaction force pairs make it possible for cars to move along a roadway surface.

## Check Your Understanding

1. While driving down the road, a firefly strikes the windshield of a bus and makes a quite obvious mess in front of the face of the driver. This is a clear case of Newton's third law of motion. The firefly hit the bus and the bus hits the firefly. Which of the two forces is greater: the force on the firefly or the force on the bus? Explain.

2. For years, space travel was believed to be impossible because there was nothing that rockets could push off of in space in order to provide the propulsion necessary to accelerate. This inability of a rocket to provide propulsion is because ...
a. ... space is void of air so the rockets have nothing to push off of.
b. ... gravity is absent in space.
c. ... space is void of air and so there is no air resistance in space.
d. ... nonsense! Rockets do accelerate in space and have been able to do so for a long time.

Explain your answer.
3. Many people are familiar with the fact that a rifle recoils when fired. This recoil is the result of action-reaction force pairs. A gunpowder explosion creates hot gases that expand outward allowing the rifle to push forward on the bullet. Consistent with Newton's third law of motion, the bullet pushes backwards upon
 the rifle. The acceleration of the recoiling rifle is ...
a. greater than the acceleration of the bullet.
b. smaller than the acceleration of the bullet.
c. the same size as the acceleration of the bullet.

Explain.
4. In the top picture (below), Kent Budgett is pulling upon a rope that is attached to a wall. In the bottom picture, Kent is pulling upon a rope that is attached to an elephant. In each case, the force scale reads 500 Newton. Kent is pulling ...

a. with more force when the rope is attached to the wall.
b. with more force when the rope is attached to the elephant.
c. the same force in each case.

## Explain.

## Identifying Action and Reaction Force Pairs

According to Newton's third law, for every action force there is an equal (in size) and opposite (in direction) reaction force. Forces always come in pairs - known as "action-reaction force pairs." Identifying and describing action-reaction force pairs is a simple matter of identifying the two interacting objects and making two statements describing who is pushing on whom and in what direction. For example, consider the interaction between a baseball bat and a baseball.


The baseball forces the bat to the left; the bat forces the ball to the right. Together, these two forces exerted upon two different objects form the action-reaction force pair. Note that in the description of the two forces, the nouns in the sentence describing the forces simply switch places.


Consider the following three examples. One of the forces in the mutual interaction is described; describe the other force in the action-reaction force pair.


Baseball pushes glove leftwards.


Bowling ball pushes pin leftwards.
$\qquad$

Enclosed air particles push balloon wall outwards.

## Check Your Understanding

1. Consider the interaction depicted below between foot A , ball B , and foot C . The three objects interact simultaneously (at the same time). Identify the two pairs of actionreaction forces. Use the notation "foot A", "foot C", and "ball B" in your statements.

$\qquad$
$\qquad$
2. Identify at least $\underline{\underline{\mathbf{i x}}}$ pairs of action-reaction force pairs in the following diagram.

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
